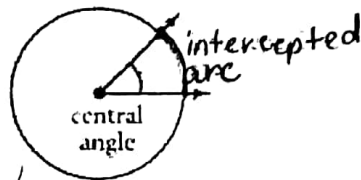
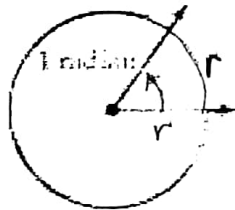


Using Radian Measure



A central angle of a circle is an angle with a vertex at the center of a circle.

An intercepted arc is the portion of the circle with endpoints on the sides of the central angle and remaining points within the interior of the angle.



When a central angle intercepts an arc that has the same length as a radius of the circle, the measure of the angle is defined to be one radian.

Because the circumference of a circle is $2\pi r$, there are 2π radians in any circle.

Since 2π radians = 360° and therefore π radians = 180° , you can use a

proportion such as $\frac{d^\circ}{180^\circ} = \frac{r \text{ radians}}{\pi \text{ radians}}$ to convert between degrees and radians.

Using a Proportion to Convert Degrees to Radians

- a. Find the radian measure of an angle of 60° .

$\frac{60}{180} = \frac{r}{\pi}$	Write a proportion.
$60\pi = 180r$	Write the cross-products.
$\frac{60\pi}{180} = r$	Divide each side by 180.
$\frac{\pi}{3} = r$	Simplify.
An angle of 60° measures $\frac{\pi}{3}$ or 1.05 radians	

$$C = 2\pi r \quad C = \pi d \quad \text{or} \quad \pi = \frac{C}{d}$$

b. Find the degree measure of $\frac{5\pi}{2}$ radians.

$\frac{d}{180} = \frac{5\pi}{2}$	Write a proportion.
$\pi d = \frac{5\pi}{2} (180)$	Write the cross-products.
$d = \frac{5}{2} (180)$	Divide each side by π .
$d = 450^\circ$	Simplify.
An angle of $\frac{5\pi}{2}$ radians equals 450°	

You Try:

a. 85° to radians.

$$\frac{d}{180} = \frac{r}{\pi}$$

$$\frac{85}{180} = \frac{r}{\pi}$$

$$\frac{85\pi}{180} = r \quad \frac{17\pi}{36} = r$$

$$1.48 = r$$

b. 2.5 radians to degrees.

$$\frac{d}{180} = \frac{r}{\pi}$$

$$\frac{d}{180} = \frac{2.5}{\pi}$$

$$d\pi = 450$$

$$d = \frac{450}{\pi}$$

$$d = 143.24^\circ$$

Converting Between Radians and Degrees

To convert degrees to radians multiply by $\frac{\pi \text{ radians}}{180^\circ}$.

To convert radians to degrees multiply by $\frac{180^\circ}{\pi \text{ radians}}$.

Example

Find the degree measure of an angle of $-\frac{3\pi}{4}$ radians.

$$-\frac{3\pi}{4} \left(\frac{180}{\pi} \right) = -135^\circ$$

You can find the Sine and cosine of angles in radian measure by using the calculator in radian mode or by using your unit circle.

Finding Cosine and Sine of Radian Measures

Find the exact values of $\cos\left(\frac{\pi}{4}\right)$ and $\sin\left(\frac{\pi}{4}\right)$ using the unit circle and approximate values using the calculator.

$$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{2}}{2} = 0.707106$$

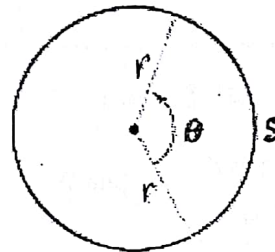
$$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

Finding the Length of an Arc

For a circle of radius r and a central angle of measure θ (in radians), the length s of the intercepted arc is:

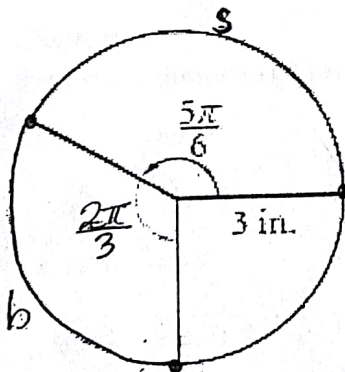
$$s = r\theta$$

\swarrow arc length \uparrow radius \nwarrow central angle



Example

Use the circle below. Find the length of s to the nearest tenth.



$$s = 3\left(\frac{5\pi}{6}\right)$$

$$s = \frac{15\pi}{6} = 7.9$$

$$b = 3\left(\frac{2\pi}{3}\right) = 2\pi = 6.3$$

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